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Quantum Effects in Friedmann Space and Creation of Macroscopic Mass

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1. INTRODUCTION

The effect of particle creation from the vacuum by an external classical field has given rise to the idea of the creation of the universe from the vacuum due to quantum processes in an intense gravitational field. Here an important question arises: what were these particles constituting the original universe? Could these particles be the usual electrons and protons, or did they have to be particles of some special kind which are not observed today?

There is also another question: if the universe was created from some gravitational foam, then if somebody could measure Planckian time intervals or do measurements on Planckian lengths in the modern epoch, would he or she see the big bang now? Here I will show how quantum effects of vacuum polarization in early Friedmann space-time give rise to an effective change of the gravitational constant which leads to the possibility of the creation of particles with mass of the order of the mass of the observable universe or some effective mass corresponding to its entropy. Then, due to the change of the gravitational constant these particles exploded as black holes and now only particles with a microscopic mass can be created from the vacuum, so there can be no big bang now. Schrödinger cats, observers, and other macroscopic bodies cannot be created from the vacuum now due to quantum processes. A long time of evolution—astrophysical and biological—is needed to create them from elementary particles. For small values of the gravitational constant one can have a noncontradictory notion of the deBroglie wave of a macrobody and in the extreme it leads to a scalar quantum field of the macroscopic universe. This field is scalar for the universe with zero angular momentum (spin).

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So if the gravitational constant is small enough, one has an epoch where "Schrödinger cats" are observable—this is the quantum era of the universe. A change of gravitational constant due to quantum effects leads to the "confinement of Schrödinger cats" inside the Planck length—the universe becomes macroscopic, and there is no sense now in the notion of scalar field—cats, trucks, stars, and the universe are not described by wave functions. So a change of gravitational constant leads to an effect similar to phase transfer in the universe—it becomes classical and macroscopic.

I discuss this question of the role of particle creation and vacuum polarization for free conformal scalar field in a Friedmann universe in the first part of this paper. In the second part I investigate the role of the self-interaction term $\lambda\varphi^4$ or $\lambda(\varphi^*\varphi)^2$ in the open Friedmann space-time for spontaneous breaking of gauge symmetry and obtain some new results for the special case of self-consistent models.

2. PARTICLE CREATION AND VACUUM POLARIZATION OF CONFORMAL MASSIVE SCALAR PARTICLES IN THE FRIEDMANN UNIVERSE

As is well known in quantum field theory, in classical nonstationary space-time, effects of particle creation from vacuum and vacuum polarization arise. The difference between these two effects is that if one thinks about nonstationary space-time in terms of an external classical field, then the effect of vacuum polarization disappears when the external field is absent, while effects of particle creation do not disappear; particles continue to exist without the external field.

The effect of particle creation can lead to the idea of quantum creation of our universe from the vacuum, so that before some moment there was the vacuum and after this moment one has a matter- or radiation-dominated universe.

Calculations of vacuum polarization and particle creation for different fields in Friedmann space-time have been given in many papers of the author together with S. G. Mamayev and V. M. Mostepanenko and are summarized in Grib *et al.* (1988*a*), so here I give only some main results which have important physical consequences.

The metric of space-time is

$$dS^2 = a^2(\eta)(d\eta^2 - d\vec{l}^{\,2}), \qquad a(\eta)\, d\eta = c\, dt$$

$$d\vec{l}^{\,2} = \lambda_{\mu\nu}\, dx^\mu\, dx^\nu = d\tau^2 + f^2(r)(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$f(r)=\begin{cases}\sinh r & x=-1\\ r & x=0\\ \sin r & x=1\end{cases} \tag{1}$$

The scalar field satisfies the equation

$$\left(\nabla^i\nabla_i+\frac{R}{6}+m^2\right)\varphi(x)=0, \qquad i=0,1,2,3 \tag{2}$$

There are two reasons for a conformal massive scalar field in Friedmann space-time: (1) only this equation is conformal invariant for the case $m=0$; (2) only the equation with an $R/6$ term corresponds to scalar particles which in the classical limit approximation move along geodesics, as must be the case in general relativity (Chernikov and Tagirov, 1988).

As it is well known (Grib *et al.*, 1988*a*), particle creation is significant in Friedmann space-time for a Compton time when the curvature is large enough so that

$$|R^{\alpha}_{0\beta 0}|\geq \ell_i^2=m^2 \tag{3}$$

where $R^{\alpha}_{0\beta 0}$ is the Riemann curvature tensor. But for the same time from the uncertainty relation $\Delta E\ \Delta t\geq h$ one has the possibility that particles with mass m can appear from the vacuum. The external field (here the gravitational field) plays two roles: of the observer registering two moments and of the energetic reservoir so that created particles are real and not virtual.

The field $\varphi(x)$ is quantized. Then the vacuum state $|0\rangle$ at some initial moment is defined as the ground state of the instantaneous Hamiltonian which is constructed by use of the metrical energy-momentum tensor for the field.

Finite expressions for the density of created particles and for vacuum expectation values $\langle 0|\hat{T}_{ik}|0\rangle$ including vacuum polarization were obtained by the method of diagonalization of the instantaneous Hamiltonian and renormalization technique for scalar, spinor, vector particles and gravitons (Grib *et al.*, 1988*a*). The method of Hamiltonian diagonalization and its use in the definition of particles is quite natural for Friedmann space-time, because the Hamiltonian used is directly connected with conformal Killing vector [there are some misleading remarks in Fulling (1979), which although correct in the general case, have nothing to do with the case of Friedmann space-time].

It is easy to understand that for time $t\ll m^{-1}$ the effective energy of the particle is large enough so that one can neglect the term m^2 in (2). But for $m=0$ the vacuum defined by us through the instantaneous Hamiltonian is conformal invariant and equation (2) is conformal invariant, too, and there is no particle creation for the massless case. That is why there is some "era of particle creation:" particles are not created by very strong gravitational field and by a very small field.

An interesting fact for conformal massive particles is that for them the vacuum expectation value of the operator of the energy-momentum tensor $\langle 0|\hat{T}_{ik}|0\rangle$ has only divergences of the type $\sim p^4$ which are subtracted by normal ordering as in the Minkowski space-time.

Yet there is some ambiguity connected with the problem of conformal anomaly. For other fields divergences divergences $\sim p^2$ and $\sim \ln p$ appear. They appear also in the anisotropic case for all fields. As is well known, subtraction of these divergences corresponds to the renormalization of the gravitational constant and the constant in the term quadratic in curvature. If one postulates a unique procedure for all cases, then one must make these subtractions for conformal particles in Friedmann space-time, too. They are finite for this case and correspond to putting "by hand" some term independent of mass m into $\langle 0|:\hat{T}_{ik}:|0\rangle$ (Grib *et al.*, 1988*a*). After this one has

$$\langle 0|:\hat{T}_{ik}:|0\rangle = T^0_{ik} + T^m_{ik} \tag{4}$$

where

$$T^0_{ik} = (1440\pi^i)^{-1}[^{(3)}H_{ik} - \tfrac{1}{6}{}^{(i)}H_{ik} - 6\delta_{x,-1}J_{ik}]$$

The term T^m_{ik} for $t \ll m^{-1}$ is

$$T^m_{ik} = \frac{m^2}{288\pi^2} G_{ik} + \frac{m^4}{128\pi^2} g_{ik} \ln\left(\frac{\mathcal{R}}{m^4}\right) \tag{5}$$

where $\mathcal{R}$ is some combination of g_{ik}, $\mathcal{R}_{ik}$ of the same dimension m^4.

For $t \gg m^{-1}$ one has particle creation:

$$\begin{gathered}\langle 0|:\hat{T}^0_0:|0\rangle \sim \frac{\text{const}}{a^3} \\ \langle 0|:T^\alpha_\alpha|0\rangle \ll \langle 0|:T^0_0:|0\rangle\end{gathered} \tag{6}$$

Terms corresponding to the conformal anomaly are (3) H_{ik} and ${}^{(i)}H_{ik}$ in (4). It is interesting to note that the conformal anomaly can be made to vanish also for other cases (other fields and anisotropic case) if one uses a trick (Grib *et al.*, 1988*b*) when calculating the integral

$$\int_0^\infty dk\, k^2 \frac{(ma)^2}{k_0^5} = \frac{1}{3} \tag{7}$$

where $k_0^2 = k^2 + m^2 a^2$.

If one chooses an "infrared cutoff" Λ_i so that

$$\int_0^\infty dk\, k^2 \frac{(ma)^2}{k_0^5} \Rightarrow \int_{\Lambda_i}^\infty dk\, k^2 \frac{(ma)^2}{k_0^5} = \frac{1}{3}\left[1 - \frac{\Lambda_i^2}{(m^2a^2(\Lambda_i^2))^{3/2}}\right] \tag{8}$$

then

$$\lim_{\Lambda_i \to 0} \lim_{m \to 0} \int_{\Lambda_i}^{\infty} dk\, k^2 \frac{(ma)^2}{k_0^5} = 0$$

This "infrared cutoff" can have the sense of an effective temperature in cases when infrared divergence appears (Grib *et al.*, 1988*b*).

Nevertheless, the problem of conformal anomaly remains despite the above remarks. As it was calculated for all known particles, the backward influence of created particles in the background metric is negligible and can be significant only for heavy particles with mass close to Planck's mass.

Here I investigate the question of backward reaction just for heavy particles. For known particles with mass $m \ll m_{pl}$ the self-consistent equation

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -\delta\pi G \langle 0 | \hat{T}_{ik} | 0 \rangle_{\text{reg}} \tag{9}$$

was solved. The main term for $t \ll m^{-1}$ was a conformal anomaly without ${}^{(i)}H_{ik}$. It was shown that only a "microuniverse" can be created from the vacuum as the solution of (9). But one can obtain a "macrouniverse" by the mechanism described in Grib (1985).

The existence of a large new parameter is needed for $a \sim 10^{-3}$ cm at $t \sim t_{pl}$. It may be achieved by the effective alteration of the gravitational constant due to vacuum polarization terms. Comparing (5) and (6), one sees that for $t \ll m^{-1}$ there was in (5) a vacuum polarization term proportional to G_{ik}, which means an effective change of the gravitational constant. There is no such term in (6). If G is the modern value of this constant, then for $t \ll m^{-1}$ one has

$$\frac{1}{8\pi\tilde{G}} = \frac{1}{8\pi G} + \frac{m^2}{288\pi^2} \equiv Z^{-1} \frac{1}{8\pi g} \tag{10}$$

If m is large enough, then $\tilde{G}$ is much smaller than G. So in an intense gravitational field creating particles one has different values of gravitational constants for periods before particle creation and after. The smallness of $\tilde{G}$ corresponds to asymptotic freedom of quantum gravity. Small $\tilde{G}$ leads to huge $\tilde{m}_{pl} \sim \tilde{G}^{-1/2}$ and to the possibility for gravity to be classical even for $t \ll t_{pl} \sim G^{1/2}$. The largest imaginable mass is that corresponding to the mass of modern universe, $m_l \sim 10^{80} m_p$, or even to the entropy $S \sim 10^{86}$, so that $m \sim 10^{86} m_{pl}$. Taking this m, one has

$$m \sim \tilde{G}^{1/2} \Rightarrow Z \sim 10^{172} \tag{11}$$

The quantum era in this scheme is from

$$\hat{t}_{pl} \sim \tilde{G}^{1/2} \ll t_{pl} = G^{1/2} \tag{12}$$

We can interpret the field as the deBroglie field of the universe. The universe can be created from vacuum due to $\Delta t\, \Delta t > h$, where Δt is much smaller than "modern" Planck time. For open space-time a solution of (9) was found in Grib *et al.* (1982) which for $t \ll m^{-1}$ in terms of η is

$$a(\eta) = \left(\frac{G}{180\pi}\right)^{1/2} (d_0 + \rho_0 \eta^2) \tag{13}$$

where d_0, ρ_0 are some numbers. For $t \gg m^{-1}$ one has

$$a(\eta) = \alpha G m (\text{ch}\ \eta - 1) \tag{14}$$

where α is some constant close to unity.

In Grib *et al.* (1982) we have a "microuniverse" the scale factor of which is such that for the Planck time the "effective length" is Planckian and the open universe becomes Minkowskian for this time. A change of the gravitational constant and a huge mass in (14) lead to the cosmological size of the universe $\sim 10^{-3}$ cm for $t \approx 10^{-43}$ sec. Our model has the following features:

1. For $m^{-1} > t > \tilde{t}_{pl} \sim \tilde{G}^{1/2}$ there is an epoch where gravitation is classical and one has Friedmann space-time with quantized conformal scalar fields with mass m. The evolution of the model is defined by the vacuum expectation value of energy-momentum tensor operator with gravitational constant different than the modern value.

2. For $t \sim m^{-1}$ there is particle creation so that for $t \gg m^{-1}$ one has the energy-momentum tensor of created particles.

3. For $t \gg t_{pl} \sim G^{1/2} \gg m^{-1}$ the deBroglie field of the universe has no sense because for any $m > m_{pl}$ the Compton wavelength will be inside the Schwarzschild radius of the particle and inside the Planck length. If one takes the opinion that quantization of gravitation in cosmology must be made through the Wheeler–DeWitt equation, so that there is no time for processes on the Planck length, no period of oscillation, no wavelength—then the very notion of deBroglie wave for macrobodies has no sense. One cannot quantize the movement of the center of mass of cats, trucks, and so on without the quantization of gravity.

So Schrödinger cats "were alive" before the change of gravitational constant but now they are not observed and cannot be created from vacuum.

4. For $t \gg t_{pl} = G^{1/2} \gg m^{-1}$ the right-hand side of (10) is "classical"—the energy density which for earlier times appeared as the energy density of some quantum particles now cannot be interpreted in the same manner. In the new universe one has particles with masses $\tilde{m} \leq m_{pl} \sim G^{-1/2}$ much smaller than m. So the right side of (10) must be interpreted as classical matter, the "constituents" of which are usual particles.

5. A change of gravitational constant could lead to mini-black holes with Planckian mass (for values of G somewhere between the initial and finite values) which explode through Hawking radiation. This can lead, due to the huge energetic desert between $m_{pl} \sim G^{-1/2}$ and $m = 10^{86} m_{pl}$, to rapid thermalization of the right-hand side of (10) and to change of entropy.

There are some features similar to inflation in this phenomenon because scale dilatation due to change of the gravitational constant has the same effect of transforming microscopic scale into macroscopic. There is some difficulty in the study of the intermediate stage between $t \ll m^{-1}$ and $t \gg m^{-1}$; there one must solve the system of equations

$$R_{ik} - \tfrac{1}{2} g_{ij} R = -8\pi G \langle 0 | \hat{T}_{ik} | 0 \rangle_{\text{reg}} = -8\pi G (T^0_{ik} + T^m_{ik})$$

$$T^m_{00} = \frac{m^2}{4\pi^2 a^2} \int_0^\infty d\lambda\, \lambda^2 \int_{\eta_0}^{\eta} d\eta^2 \frac{d(a^2)}{d\eta^2} \left[|g_\lambda(\eta^i)|^2 - \frac{1}{20(\eta^i)} \right]$$

$$\tilde{g}_\lambda(\eta) + w^2(\eta) g_\lambda(\eta) = 0$$

$$w^2(\eta) = \lambda^2 + m^2 a^2(\eta)$$

For particles with spin one-half and one, one must make infinite regularization of gravitational constant for $t \ll m^{-1}$ and there is a finite term of a form similar to the scalar case for $t \gg m^{-1}$. One can take as the value of the renormalized gravitational constant the same G as for the scalar particles.

6. There is an answer in the given scenario to the question of why the universe cannot originate from vacuum (gravitational foam) for the interval Δt_{pl} by quantum creation in the modern epoch. The modern value of the gravitational constant leads to the possibility of creation of only "microuniverses" from the gravitational foam if one considers Planck intervals of time and space now.

7. If there are N internal degrees of freedom (N sorts), then

$$\frac{m^2}{288\pi^2} G_{00} \Rightarrow \frac{Nm^2}{288\pi^2} G_{00}$$

so that m, while being very close, is not equal to the Planck mass $\tilde{m}_{pl}$.

3. SPONTANEOUS BREAKDOWN OF GAUGE AND DISCRETE SYMMETRY

It was shown (Grib *et al.*, 1988*a*) in the Friedmann universe of the open type, without any tricks such as bare tachyons with $m^2 < 0$ as in the Higgs model, that one has a breakdown of gauge symmetry, which has a geometrical origin. The self-interacting scalar conformal field in the isotropic hyperbolic metric $x = -1$ obeys the equation

$$\nabla^i \nabla_i \varphi(x) + \left(m^2 + \frac{R}{6}\right) \varphi(x) + \lambda \varphi^{\alpha}(x) \varphi^{\eta}(x) = 0 \tag{15}$$

The vacuum expection value of the quantized field $\hat{\varphi}$ is

$$\langle 0 | \hat{\varphi}(x) | 0 \rangle = g(\eta) \tag{16}$$

and in the tree approximation it satisfies the equation

$$\tilde{g} + \frac{2\tilde{a}}{a} g + \left(m^2 a^2 - 1 + \frac{\tilde{a}}{a}\right) g + \lambda a^2 g^3 = 0 \tag{17}$$

Taking

$$g(\eta) = \frac{1}{\sqrt{\lambda}} \frac{f(\eta)}{a(\eta)}$$

one has

$$f + (m^2 a^2 - 1) f + f^3 = 0 \tag{18}$$

The nontrivial solution of this equation $f \neq 0$ means spontaneous breakdown of gauge symmetry and it is preferable to the trivial one if the term $(m^2 a^2 - 1)$ has negative sign. It is natural to take $a(\eta)$ as the solution of the Einstein equation, the right-hand side of which is defined by the energy-momentum tensor of the field $g(\eta)$.

For $t \ll m^{-1}$ one has the approximate solution of Einstein equations

$$a(\eta) = f \overline{\frac{\varphi \pi G}{\lambda}} \text{ ch } \eta$$

corresponding to negative energy

$$\xi(\eta) = -\frac{3}{8\lambda g^2(\eta)}$$

lower than that for the trivial solution for time $t < m^{-1}$. For $t \gg m^{-1}$ the trivial solution becomes preferable.

Calculating particle creation in the metric obtained through spontaneous breakdown of symmetry, one comes to the picture described in Section 1 of this paper.

But what is the gauge symmetry broken in the open universe? If one takes φ as the deBroglie field of the universe, one can say that it has nonzero baryon charge—so spontaneous breakdown of this symmetry can give an answer for the baryon asymmetry of the universe.

From equation (18) one can see that even without a self-interaction term there is nontrivial solution: so breaking of gauge symmetry for some time is the general situation for the open universe.

The solution $a(\eta)$ due to spontaneous breaking of symmetry can be taken instead of (13) as the external gravitational field creating particles according to mechanism of Section 2. The results will be qualitatively the same.

A final remark is about closed Friedmann space-time. It is for closed Friedmann space-time that the entropy of the world has exact sense. In order to have some classical Friedmann space-time with the scale factor of the form (13) where particles of macroscopic mass are created and entropy arises due to their Hawking explosion, one can take the approach of Halliwell and Hawking (1985). One must take some solution of the Wheeler–DeWitt equation which has quasiclassical form for some $a \neq 0$, $\varphi \neq 0$, so that time appears in this quasiclassical limit. From this point of view, $\varphi \neq 0$ corresponding to spontaneous breaking of symmetry, is necessary to have the notion of time.

Here I have not discussed the problem of singularity and the origin of classical space-time. Quantization of gravity is necessary for an answer to this question. In this paper I have treated only the possibility of matter creation from classical gravity.

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